Dynamics of fluctuation-dominated phase ordering: Hard-core passive sliders on a fluctuating surface

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We study the dynamics of a system of hard-core particles sliding downwards on a one-dimensional fluctuating interface, which in a special case can be mapped to the problem of a passive scalar advected by a Burgers fluid. Driven by the surface fluctuations, the particles show a tendency to cluster, but the hard-core interaction prevents collapse. We use numerical simulations to measure the autocorrelation function in steady state and in the aging regime, and space-time correlation functions in steady state. We have also calculated these quantities analytically in a related surface model. The steady-state autocorrelation is a scaling function of t/L^z, where L is the system size and z is the dynamic exponent. Starting from a finite intercept, the scaling function decays with a cusp, in the small argument limit. The finite value of the intercept indicates the existence of long-range order in the system. The space-time correlation, which is a function of r/L and t/L^z, is nonmonotonic in t for fixed r. The aging autocorrelation is a scaling function of t_1 and t_2 where t_1 is the waiting time and t_2 is the time difference. This scaling function decays as a power law for $t_2 \gg t_1$; for $t_1 \gg t_2$, it decays with a cusp as in steady state. To reconcile the occurrence of strong fluctuations in the steady state with the fact of an ordered state, we measured the distribution function of the length of the largest cluster. This shows that fluctuations never destroy ordering, but rather the system meanders from one ordered configuration to another on a relatively rapid time scale.

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I. INTRODUCTION

The concentration of a substance advected by a driving field such as a fluid flow often shows interesting behavior. In examples such as smoke dispersed in air or fluorescent dye carried by a turbulent jet, an initial local concentration of the passive particles generally spreads out in space [1]. However, if the fluid is compressible, instead of spreading out, the advected substance may show a tendency to cluster, as with air bubbles in water or dust particles in air [2,3].

In the situations discussed above, the advected substance has a negligible effect on the fluid flow. These problems are examples of "passive scalar" problems, where the dynamics of a nonequilibrium driving field strongly affects that of the other (passive) field with no back effect from the latter. In this paper, we study clustering in a set of passive particles which are subjected to a fluctuating force field. The specific system we study consists of hard-core particles sliding under gravity on a one-dimensional fluctuating interface; the instantaneous force on a particle is then proportional to the local slope of the surface. Interestingly, our results also pertain to particles advected by a fluid, using the fact that the equation governing a moving interface can be mapped onto the Burgers equation, which describes a compressible fluid [4].

The degree of clustering of the particles depends strongly on the interactions between them. Two cases have been studied earlier—particles which are completely noninteracting [5,6] and particles which interact via hard-core exclusion [7,8]. We study the latter, more realistic, case in this paper and address two broad issues, namely dynamics and ordering.

The first issue concerns the time-dependent properties of the passive particles. We obtain results for the autocorrelation and space-time correlation functions in steady state, and aging correlations in the approach to steady state. To our knowledge, the dynamics of passive scalars has not been explored systematically, and our study adds to the relatively sparse work on this important question [9]. We have considered two kinds of surface evolutions—namely those that respect symmetry under reflection (Edwards-Wilkinson type) and those which break this symmetry (Kardar-Parisi-Zhang type). Our simulation results for the dynamics of the sliding particle system are supplemented by analytical calculations for a related model of coarse-grained interface variables.

The second issue concerns the characterization of the steady state as an ordered state. Earlier studies of static properties revealed that strong fluctuations are present in the steady state and do not decrease even in the thermodynamic limit [7]. On the other hand, the scaling function for the density correlation indicates that the system has long-range order. This sort of fluctuation-dominated phase ordering (FDPO) is characterized by a broad distribution of the order parameter. The question then arises: In what sense does FDPO represent an ordered state, if strong fluctuations drive it between macroscopically different configurations on a relatively rapid time scale? We address this by studying the variation of the length of the largest particle cluster present in the system and show that the corresponding probability distribution provides an unequivocal signal of ordering.

II. OVERVIEW

A. Surface fluctuation and particle movement

A surface with no overhangs is completely specified by the height $h(x,t)$ at point x at time t. The evolution of the height field is taken to be described by the Kardar-Parisi-Zhang (KPZ) equation [4]:

$$\frac{\partial h}{\partial t} = \nu_1 \frac{\partial^2 h}{\partial x^2} + \lambda \left(\frac{\partial h}{\partial x}\right)^2 + \eta_1(x,t). \tag{1}$$

The first term represents the smoothening effect of surface tension ν_1, and $\eta_1(x,t)$ is a white noise with zero average and $\langle \eta_1(x,t)\eta_1(x',t')\rangle = \Gamma\delta(x-x')\delta(t-t')$. Notice that if $\lambda \neq 0$, $h \to -h$ symmetry is not preserved, reflecting the fact that the interface moves in a preferred direction. However, if $\lambda=0$, the equation has an $h\to -h$ symmetry and describes the Edwards-Wilkinson (EW) model [10].

The height-height correlation function has a scaling form for large separations in space and time [11]:

$$\langle [h(x,t) - h(x',t')]^2 \rangle \approx |x-x'|^{2\chi} f\left(\frac{|t-t'|}{|x-x'|^z}\right). \tag{2}$$

Here f is a scaling function and χ and z are the roughness and dynamic exponents, respectively, with values which depend on the surface dynamics. For an EW interface $\chi=\frac{1}{2}$, $z=2$ while for a KPZ interface $\chi=\frac{1}{2}$, $z=\frac{3}{2}$.

The hard-core particles slide downwards along the local slope $(\partial h/\partial x)$ of the interface. In the overdamped limit, their velocity is proportional to the local gradient of height. The equation governing the evolution of particle density can be derived from the continuity equation $\partial\rho(x,t)/\partial t = -\partial J(x,t)/\partial x$. The local current $J(x,t)$ has a systematic part $\rho(1-\rho)(-\partial h/\partial x)$ (which represents the current flowing down the slope, from an occupied site to a neighboring empty site), a diffusive part $-\nu_2 \partial\rho/\partial x$ (driven by local density inhomogeneity) and a stochastic part $\eta_2(x,t)$ (a Gaussian white noise). The time-evolution equation for the density fluctuation $\tilde{\rho}=\rho-\rho_0$ is then

$$\frac{\partial \tilde{\rho}}{\partial t} = \nu_2 \frac{\partial^2 \tilde{\rho}}{\partial x^2} + 2\rho_0(1-\rho_0)\frac{\partial^2 h}{\partial x^2} - (1-2\rho_0 - 2\tilde{\rho})\left(\frac{\partial \tilde{\rho}}{\partial x}\right) \times \left[1 - 2\left(\frac{\partial h}{\partial x}\right)\right] + 2(1-2\rho_0)\tilde{\rho}\frac{\partial^2 h}{\partial x^2} - 2\tilde{\rho}^2 \frac{\partial^2 h}{\partial x^2} + \frac{\partial \eta_2(x,t)}{\partial x}. \tag{3}$$

We will not analyze this equation directly; rather we will study the particle dynamics by performing numerical simulations on a lattice model, described in Sec. III, whose long distance and long time properties are expected to be described by Eqs. (1) and (3).

B. FDPO: Static properties

In an earlier study on static properties of this model [7], the density-density correlation $C(r,L)$ of the sliding particles was measured and found to be a scaling function of r/L in the scaling limit $r\to\infty$, $L\to\infty$ with r/L fixed, as for phase-ordered states. The scaling function has a finite intercept m^2, and for small argument it decays with a cusp:

$$C(r,L) = f\left(\frac{r}{L}\right) \tag{4}$$

$$= m^2\left[1 - a\left(\frac{r}{L}\right)^\alpha\right], \quad \frac{r}{L} \ll 1. \tag{5}$$

m^2 is a measure of long-range order (LRO) as LRO is defined by the large r behavior of the correlation function, with $r \ll L$. In the limit of an infinite system, this corresponds to $r/L \to 0$.

The value of the cusp exponent α depends on the dynamics of the driving surface: $\alpha \simeq 0.5$ for particles on an EW interface $(\lambda=0)$ while $\alpha\simeq 0.25$ when the underlying surface is of KPZ type, with either sign of λ [7]. In a related coarse-grained surface model defined in [7] and discussed in Sec. III below, the correlation function was shown analytically to have the above scaling form with $m^2=1$ and $\alpha=0.5$ for both EW and KPZ surfaces. For hard-core particles sliding on a two-dimensional surface, the same scaling form is found for $C(r,L)$, but with a different value of the intercept and the cusp exponent. For customary phase-ordering systems, it is expected that this scaling function should decay linearly, consistent with the Porod law [13]. The cusp $(\alpha<1)$ is one manifestation of the unusual nature of FDPO.

For a phase-ordered system, the lowest nonzero Fourier component of the density profile measures the extent of phase-separation and is an appropriate order parameter. A characteristic of FDPO is that the distribution of this order parameter remains broad even in the thermodynamic limit, indicating the presence of strong fluctuations.

C. FDPO: Dynamical properties

In this paper, we study the dynamics associated with FDPO. Our results are summarized below. We find that the steady-state autocorrelation in the density fluctuation $\sigma(x,t)$ of the sliding particles (SP): $A_{SP}(t)=\langle\sigma(x,0)\sigma(x,t)\rangle$ is a scaling function of t/L^z, consistent with the notion of phase ordering. For $t \ll L^z$, this scaling function decays with a cusp, in contrast to a linear decay normally expected for phase-ordering systems. The presence of the cusp is the dynamical manifestation of the unconventional character of FDPO.

We also monitored the autocorrelation function in the aging regime $\mathcal{A}_{SP}(t_1,t_2)=\langle\sigma(x,t_1)\sigma(x,t_1+t_2)\rangle$. From the dynamic scaling hypothesis, $\mathcal{A}_{SP}(t_1,t_2)$ should be a scaling function of t_1/t_2, the ratio of initial time to the time lag [12]. In the limit $t_2 \gg t_1$, the scaling function shows a power law decay with an exponent that depends on the phase-ordering kinetics. In the opposite limit $t_1 \gg t_2$, the scaling function decays with a cusp as in steady state, the only difference being that the system size L, as appears in the steady-state scaling function, is replaced by the coarsening length $t_1^{1/z}$.

The space-time correlation for the sliding particles in steady state, defined as $G_{SP}(r,t)=\langle\sigma(x,0)\sigma(x+r,t)\rangle$, is a function of r/L and t/L^z. When plotted against t/L^z for a fixed value of r/L, this function is nonmonotonic and decays like the steady-state autocorrelation for large t. The autocorrelation function in steady state and in the aging regime, together with the space-time correlation function in steady state, constitute our dynamical characterization of FDPO.

D. FDPO: An ordered state?

The distinctive feature of FDPO is the presence of strong fluctuations in steady state. On the one hand, a diagnostic of LRO such as a nonzero value of the intercept m^2 indicates an ordered state. On the other hand, the macroscopic state of the system changes relatively rapidly over a time scale $\sim L^z$, in contrast to a normal phase-ordered system, where the typical time grows exponentially with L. This relatively small lifetime of a given macroscopic ordered state may appear surprising and seem to contradict the notion of LRO. To resolve this apparent contradiction we study the time dependence of the length of the largest particle cluster $l_{max}(t)$. We conclude that the short lifetime is associated with the system wandering over a multitude of ordered states, each very different from the other, but all characterized by a large value of $l_{max}(t)$. Dynamical excursions away from this attractor of ordered states are extremely infrequent and associated with an exponentially growing time scale.

III. DESCRIPTION OF THE MODEL

We study a discrete model of a fluctuating interface on which hard-core particles slide downwards under gravity, following the local slope of the interface. The 1-d interface of length L consists of discrete surface elements; the slope of the surface elements between the ith and $(i+1)$th site is $\tau_{i+1/2}$, which can take the value +1 or −1. Accordingly the height at site i is given by $h_i=\Sigma_{j=1}^{i}\tau_{j-1/2}$. The dynamics follows that of the single-step model [14,15] which involves stochastic corner flips with exchange of adjacent τ's; the transition $\wedge$ to $\vee$ occurs with a rate p_1 while $\vee$ to $\wedge$ with rate q_1. As in [14,15], we take $p_1=q_1=1$ to represent an EW surface and $p_1=1$, $q_1=0$ for a KPZ surface. The overall slope $\mathcal{T}=(1/L)\Sigma_{i=1}^{L}\tau_{i+1/2}$ is conserved and in our case we will consider $\mathcal{T}=0$, meaning that the interface is untilted.

The hard-core particles are represented by variables $\{\sigma_i\}$, each of which takes a value +1 or −1 according as the ith site contains a particle or a hole. The deviation from half-filling $\mathcal{S}=(1/L)\Sigma_{i=1}^{L}\sigma_i$ is conserved. A particle and hole on adjacent sites $(i,i+1)$ exchange with rates that depend on the intervening local slope $\tau_{i+1/2}$; thus the moves $\bullet\backslash\circ\to\circ\backslash\bullet$ and $\circ/\bullet\to\bullet/\circ$ occur at rate p_2 while the inverse moves occur at rate q_2. In the case when the particles are sliding downwards along gravity, we have $q_2<p_2$. We have considered $p_2=1$ and $q_2=0$. Because of the hard-core exclusion between the particles, in a half-filled system with the above update rules, one has particle-hole symmetry, i.e., any correlation function involving the density variable remains invariant when the particle density is replaced by the hole density. This implies that the correlation measured in an advection process (with $\lambda>0$, when the surface is moving downwards, along the same direction as the particles) is exactly the same as that in an antiadvection process ($\lambda<0$, i.e., the surface moves upwards, opposite to the particle movement). This is an important difference from the case of noninteracting particles, where the correlations in advection and antiadvection show qualitatively different behavior.

From the dynamical rules, it follows that the movement of particles depends on the fluctuations of the underlying interface. Due to gravity the particles tend to slide down into local valleys. However, in the nonequilibrium system under consideration, before the particles can fill in the lowest valley, the interface evolves, often causing the valley to turn over. Nevertheless, it is useful to consider the adiabatic limit where the interface moves infinitely more slowly than the particles, in which case the particles have ample time to explore the landscape and eventually settle in the deepest valleys. It seems plausible that the dynamics of hills and valleys of the interface may provide insight into the dynamics of the particles. This motivates the definition of a coarse-grained depth model (CD model) as follows [7]. Consider the variable $s_i(t)$ defined as $s_i(t)=\mathrm{sgn}[h_i(t)-\langle h(t)\rangle]$, where $\langle h(t)\rangle$ is the average height at time t: $\langle h(t)\rangle=(1/L)\Sigma_{i=1}^{L}h_i(t)$. The variable $s_i(t)$ can take values +1, −1, or 0, depending on whether the position of the ith site is above, below, or at the average level. In other words, $s_i(t)$ gives a coarse-grained description of the surface by labeling "highlands" and "lowlands." For an EW interface, the dynamics is tractable and we obtain an analytic expression for time-dependent correlations of $s_i(t)$. These results might be expected to be close to those of $\sigma_i(t)$ in the extreme adiabatic limit. As a matter of fact, we find that they also describe qualitatively the particle model even in the strongly nonequilibrium case.

IV. AUTOCORRELATION FUNCTION IN STEADY STATE

We have studied the autocorrelation $A(t,L)$ involving the density variable $\langle\sigma_i(0)\sigma_i(t)\rangle$ in the sliding particle (SP) model and also that for the CD variables $\langle s_i(0)s_i(t)\rangle$ in the CD model. We have considered a half-filled system ($\mathcal{S}=0$) with periodic boundary conditions. We will see below that in the steady state of a system of size L, the autocorrelation $A(t,L)$ is a scaling function of t/L^z, where z is the surface dynamic exponent defined earlier. Since the particles try to settle in the valleys, the time scale of the decay of the density autocorrelation is expected to be of the order of the lifetime $\sim L^z$ of a large valley. This scaling function shows a cusp in the small argument limit, as seen previously in the static correlation scaling function [Eq. (5)]:

$$A(t,L)=h\left(\frac{t}{L^z}\right) \tag{6}$$

$$=m^2\left[1-b\left(\frac{t}{L^z}\right)^{\beta'}\right],\quad \frac{t}{L^z}\to 0. \tag{7}$$

m is a measure of the LRO as explained in Sec. II. Note that m^2 is the value reached by the autocorrelation function at a large enough but finite (L-independent) time.

However, for small time, $t\lesssim 1$, which falls outside the scaling regime, the autocorrelation function shows a linear drop with an L-dependent slope:

TABLE I. The values of relevant exponents and intercepts for dynamical characterization of CD model and SP model with $\mathcal{S}=0$.

	CD model		SP model	
	EW	KPZ	EW	KPZ
m^2	1.0	1.0	0.82±0.03	0.75±0.04
β'	0.25	0.31±0.02	0.22±0.02	0.18±0.01
δ	0.5	0.5	0.26±0.005	0.15±0.005
γ	0.75	0.84±0.02	0.69±0.02	0.82±0.04

$$A(t,L) \approx 1 - b' \frac{t}{L^{\delta}}, \quad t \lesssim 1. \tag{8}$$

If $m^2=1$, as shown below for the CD model, matching Eqs. (7) and (8) for $t \simeq 1$ yields

$$\delta = z\beta'. \tag{9}$$

If $m^2 \neq 1$, as happens for the SP model, a relation between the exponents cannot be obtained. Instead, the matching condition determines a time scale t^* for the crossover from the linear decay in Eq. (8) to the cuspy decay in Eq. (7). In the large L limit, we find, to the leading order,

$$t^* = \frac{1-m^2}{b'} L^{\delta}. \tag{10}$$

We have summarized the values of the intercept and the exponents in Table I, for the CD model and the SP model on EW and KPZ surfaces.

Let us illustrate these properties, by discussing the autocorrelation in the CD model, defined as $A_{CD}(t,L) = \langle s_i(0)s_i(t)\rangle$. First consider short times $t \lesssim 1$. At $t=0$ let the initial configuration of the surface be $\{h_i(0)\}$. As time passes, there are stochastic corner flips, as described in Sec. III. However, only those flips occurring close to the average level can cause a change in the CD variable $s_i(t)$, as any local fluctuation far above or below the average level would not change the sign of $s_i(t)=[h_i(t)-\langle h(t)\rangle]$. More precisely, only those sites in $\{h_i(0)\}$, which have at least one neighbor situated exactly on the average level, putatively contribute to the drop in autocorrelation function. Now, for a self-affine surface of length L and roughness exponent χ, the number of such points scales as $L^{1-\chi}$ and the density of such points goes as $L^{-\chi}$ [16]. For small t, the probability that any one of these points will actually take part in a local fluctuation is proportional to t. This immediately implies $A_{CD}(t \lesssim 1, L) \approx 1 - b_1'(t/L^{\chi})$. Comparison with Eq. (8) shows that for the CD model, we have $\delta=\chi=\frac{1}{2}$. Note that although EW and KPZ surfaces have different dynamics, the above argument holds for both of them as their stationary measure is the same in 1-d.

For the particle model, although the initial drop is found to be linear as described in Eq. (8), the exponent δ takes the value 0.26±0.005 for particles on an EW surface and 0.15±0.005 for particles on a KPZ surface. The data are shown in Fig. 1.

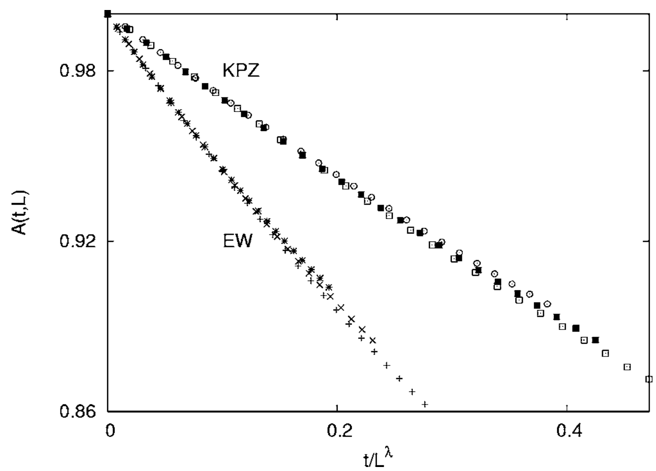

FIG. 1. Illustrating the linear drop of $A(t,L)$ for short times $t \lesssim 1$ in the SP model for system size $L=128, 256, 512$.

For $t \gtrsim 1$, we have analytically calculated $A_{CD}(t,L)$ for an EW interface. This exploits the fact that $h_i(t)$ in this case is a Gaussian variable, implying s_i correlations satisfy the arcsine law

$$\langle s_i(t)s_i(0)\rangle = \frac{2}{\pi}\sin^{-1}\left(\frac{\langle H_i(t)H_i(0)\rangle}{\sqrt{\langle H_i^2(t)\rangle\langle H_i^2(0)\rangle}}\right) \tag{11}$$

where $H_i(t)=h_i(t)-\langle h(t)\rangle$, which is also a Gaussian variable. If $\tilde{h}_k(t)$ is the Fourier transform of $h_i(t)$, the numerator in the argument of arcsine can be written as $\Sigma_{k\neq 0}\langle \tilde{h}_k(t)\tilde{h}_{-k}(0)\rangle = \Sigma_{k\neq 0}\Gamma \exp(-c_k t)/c_k$, using the discrete version of the EW equation. Here, $c_k = 4\nu_1 \sin^2 k/2$. Moreover, $\langle H_i^2(t)\rangle = \langle H_i^2(0)\rangle = \Gamma \Sigma_{k\neq 0} 1/c_k$. Thus we have

$$\langle s_i(t)s_i(0)\rangle = \frac{2}{\pi}\sin^{-1}\left[\frac{\sum_{k\neq 0}\frac{\exp(-c_k t)}{c_k}}{\sum_{k\neq 0}\frac{1}{c_k}}\right] \tag{12}$$

We have numerically evaluated this discrete sum and plotted it in Fig. 2(a) against the scaling argument t/L^2 for different L values. The cusp exponent can be read off from the plot in the inset.

In the continuum limit, Eq. (12) becomes

$$\langle s(x,t)s(x,0)\rangle = \frac{2}{\pi}\sin^{-1}\left[\frac{\int_{2\pi/L}^{\pi} dk \frac{\exp(-k^2 t)}{k^2}}{\int_{2\pi/L}^{\pi}\frac{dk}{k^2}}\right]. \tag{13}$$

The integral in the numerator takes the form

$$\frac{L\Gamma}{\pi}\left[\frac{L}{2\pi}\exp\left(-\frac{4\pi^2 t}{L^2}\right) + \sqrt{\pi t}\, erf\left(\frac{2\pi\sqrt{t}}{L}\right) - \sqrt{\pi t}\right].$$

In the limit $t/L^2 \ll 1$, this becomes, to the leading order, $(L\Gamma/\pi)(L/2\pi - \sqrt{\pi t})$. Noting that the denominator is

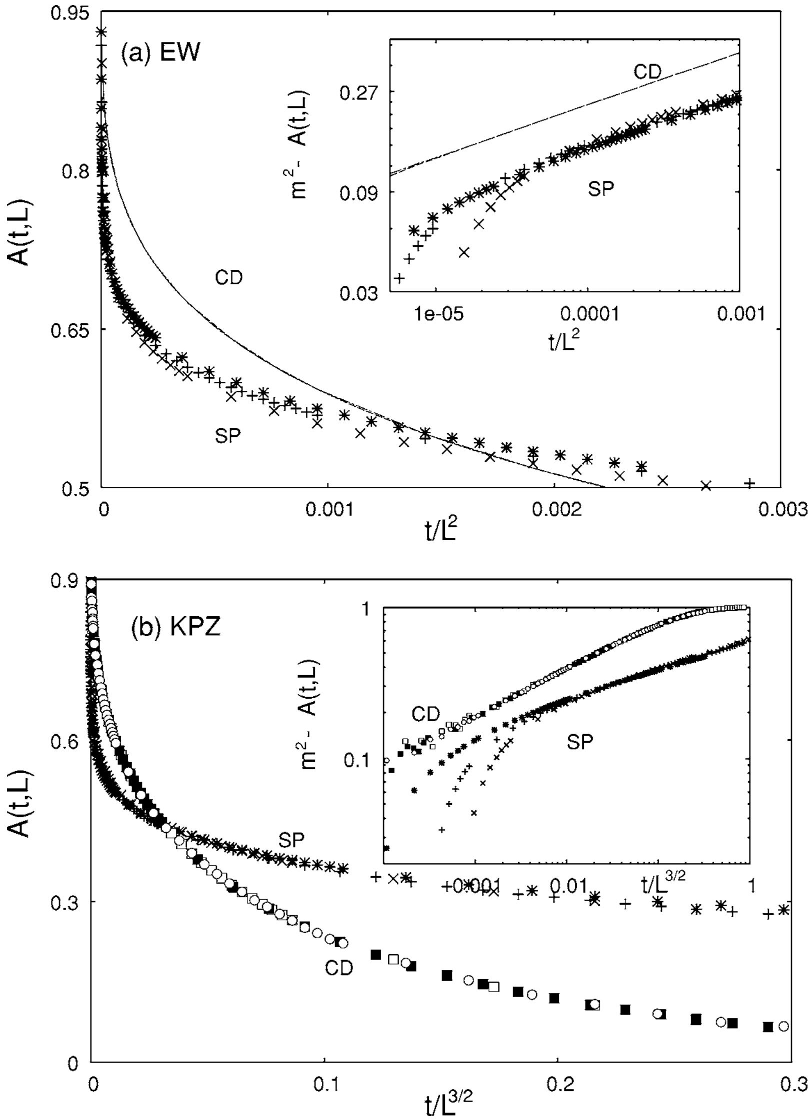

FIG. 2. Scaled autocorrelation function in steady state for SP and CD models for (a) EW and (b) KPZ interfaces. In both cases, we used $L=512,1024,2048$. The cusp exponents were estimated using the plots shown in the inset.

$L^2\Gamma/2\pi^2$ and expanding for small values of $\sqrt{t}/L$, we get

$$\langle s(x,t)s(x,0)\rangle \approx 1 - \frac{4}{\pi^{1/4}}\left(\frac{t}{L^2}\right)^{1/4}, \quad (t/L^2 \ll 1). \qquad (14)$$

Comparing with Eq. (7) gives $m^2=1$, $\beta'=\frac{1}{4}$, $z=2$.

For the KPZ surface, the time evolution equation for the height field is not Gaussian and hence such an analytical treatment is not possible. We study $A_{CD}(t)$ using Monte Carlo simulation. No initial equilibration is required as the steady-state measure for a KPZ surface with periodic boundary conditions gives equal weight to every configuration. The initial configuration was thus chosen randomly. We followed the update rules discussed in Sec. II and averaged over sites as well as over 10^5 histories. The results are shown in Fig. 2(b). A good scaling collapse is obtained for different L, on resscaling the time to t/L^z with $z=\frac{3}{2}$. The cusp exponent β' was extracted by plotting $m^2-A_{CD}(t)$ against t/L^z (shown in the inset), using $m^2=1$ and this gives β' to be 0.31 ± 0.02. Our best estimate corresponds to the largest system size $L=2048$. The error bar is based on the values of β' obtained for smaller system size ($L=512,1024$); the statistical error is much smaller.

For the sliding particle (SP) model, the steady-state measure is not known analytically. In our simulation, we started from a randomly disordered configuration and allowed a long time, $\sim 10L^z$, for the system to reach a steady state. We then measured $(1/L)\Sigma_{i=1}^{L}\sigma_i(0)\sigma_i(t)$ for approximately L^z time steps. We waited several thousand time steps before repeating the procedure and averaged over 10^4 histories.

For particles sliding on an EW interface, we obtained a good scaling collapse of $A_{SP}(t,L)$ for different L after rescaling time to t/L^2 [Fig. 2(a)]. The cusp exponent was extracted by fitting $m^2-A_{SP}(t,L)$ to a power law. We have estimated m^2 by using the same technique as discussed in [7]. The best estimate of m^2 corresponds to the value for which the structure factor has the largest power law stretch. We found that m^2 shows a systematic dependence on L and the cusp exponent β' is in fact quite sensitive to the value of m^2. We have used $m^2\simeq 0.82$, our estimate from the largest system size we could access ($L=4096$). This yields $\beta'\simeq 0.22$. On the other hand, using $m_\infty^2\simeq 0.85$, which we get by extrapolating the dependence of m^2 on L for an infinite system, we find $\beta' \simeq 0.20$.

For the SP model on a KPZ surface, we find $m^2 \simeq 0.75$. Figure 2(b) shows the scaling collapse for different L after rescaling the time by $L^{3/2}$. The inset shows that $m^2 - A_{SP}(t,L)$ follows a power law and the exponent is found to be $\beta' \simeq 0.18$. The value of β' obtained using m_∞^2 is $\simeq 0.17$.

Apart from the half-filled case, we have also studied the autocorrelation function for filling fractions $\frac{1}{4}$ and $\frac{1}{8}$ (corresponding to $\mathcal{S}=-\frac{1}{2}$ and $\mathcal{S}=-\frac{3}{4}$, respectively). We found that the same scaling form [Eq. (7)] holds. However, the value of the intercept changes while the cusp exponent remains the same.

V. AUTOCORRELATION IN AGING REGIME

The aging autocorrelation function $\mathcal{A}(t_1,t_2)$ is defined as $\langle \sigma_i(t_1)\sigma_i(t_1+t_2)\rangle$ for the particles and as $\langle s_i(t_1)s_i(t_1+t_2)\rangle$ for the CD variables. We have investigated primarily the half-filled case, but have checked that no qualitative change takes place for other values of filling fraction. $\mathcal{A}(t_1,t_2)$ depends on both t_1 and t_2. For $1 \ll t_1, t_2 \ll L^z$, $\mathcal{A}(t_1,t_2)$ is a function of t_1/t_2, as expected for phase-ordering systems [12]. In the limit when $t_2 \gg t_1$, this scaling function has a power law decay (see Table I)

$$\mathcal{A}(t_1,t_2) \sim \left(\frac{t_1}{t_2}\right)^{\gamma} \quad \text{for } t_2 \gg t_1, \tag{15}$$

while in the opposite limit, $t_1 \gg t_2$, the scaling function has the form

$$\mathcal{A}(t_1,t_2) \sim m^2\left[1 - b_1\left(\frac{t_2}{t_1}\right)^{\beta'}\right] \quad \text{for } \frac{t_2}{t_1} \to 0. \tag{16}$$

This is similar to the form of the steady-state autocorrelation in Eq. (7) with L replaced by $t_1^{1/z}$, meaning that locally the system has reached steady state over a length scale of $t_1^{1/z}$.

We first present our results on the CD model. As in the case of steady-state autocorrelation, we have been able to calculate $\mathcal{A}_{CD}(t_1,t_2)$ for an EW surface analytically. Following similar steps as in the last section, we obtain

$$\mathcal{A}_{CD}(t_1,t_2) = \frac{2}{\pi}\sin^{-1}\left(\frac{\displaystyle\sum_{k\neq 0}\frac{\exp(-c_k t_2) - \exp[-c_k(2t_1+t_2)]}{c_k}}{\left(\displaystyle\sum_{k'\neq 0}\frac{1-\exp(-2c_{k'}t_1)}{c_{k'}}\right)^{1/2}\left(\displaystyle\sum_{k''\neq 0}\frac{1-\exp[-2c_{k''}(t_1+t_2)]}{c_{k''}}\right)^{1/2}}\right). \tag{17}$$

Taking the continuum limit and using $t_1, t_2 \ll L^2$, we obtain

$$\mathcal{A}_{CD}(t_1,t_2) = \frac{2}{\pi}\sin^{-1}\left(\frac{\sqrt{2t_1+t_2}-\sqrt{t_2}}{(2t_1)^{1/4}(2t_1+2t_2)^{1/4}}\right). \tag{18}$$

In the limit $t_2 \gg t_1$, the right-hand side becomes $(\sqrt{2}/\pi) \times (t_1/t_2)^{3/4}$. Comparing with Eq. (15), we get $\gamma = \frac{3}{4}$. In the opposite limit, when $t_1 \gg t_2$, the right-hand side becomes, after simplification,

$$\mathcal{A}_{CD}(t_1,t_2) \approx 1 - \frac{2^{5/4}}{\pi}\left(\frac{t_2}{t_1}\right)^{1/4}. \tag{19}$$

Comparing with Eq. (16), we find $\beta' = \frac{1}{4}$, as expected.

Figure 3(a) shows the numerical evaluation of the discrete sum in Eq. (17). The power law characterizing the decay has been shown in the inset.

In our Monte Carlo simulations, we have a spatial average as well as an average over 10^4 histories. For the CD model of a KPZ surface, we started with a flat interface as an initial condition and evolved it in time to measure $\mathcal{A}_{CD}(t_1,t_2)$. The results are shown in Fig. 3(b). The best estimate of the cusp exponent corresponds to $t_1 = 32\,000$ and the error bar is based on its values for $t_1 = 2000, 8000$. This finally gives $\beta' = 0.31 \pm 0.01$, which is close to the steady-state value. The inset shows the power law decay and the exponent γ takes the value 0.84 ± 0.03. Here, the best estimate is for $t_1 = 500$ and the error bar is for $t_1 = 2000, 8000$.

For the SP model on an EW interface, we start with randomly distributed particles on a random surface profile. The aging autocorrelation $\mathcal{A}_{SP}(t_1,t_2)$ shows a scaling collapse as plotted against t_2/t_1 [see Fig. 3(a)]. The value of the cusp exponent β' is 0.20 ± 0.02, close to its steady-state value. The inset shows plot in the regime $t_2 \gg t_1$. The power law exponent in this case is $\gamma = 0.69 \pm 0.02$.

The SP model on a KPZ surface also starts with the random initial condition. The data are shown in Fig. 3(b). The exponents are $\beta' = 0.17 \pm 0.01$ and $\gamma = 0.82 \pm 0.04$.

VI. SPACE-TIME CORRELATION IN STEADY STATE

In this section, we discuss the behavior of space-time correlation $G(r,t,L)$ defined in steady state as $\langle \sigma_i(0)\sigma_{i+r}(t)\rangle$ for the particles and $\langle s_i(0)s_{i+r}(t)\rangle$ for the CD variable. $G(r,t,L)$ does not show any L-independent scaling between r and t. Rather, it is a function of the scaled variables $\xi = r/L$ and $\tau = t/L^z$:

$$G(r,t,L) = g(\xi,\tau). \tag{20}$$

With ξ held fixed, g shows an interesting nonmonotonic behavior with τ. $g(\xi,0)$ reduces to the pair correlation function

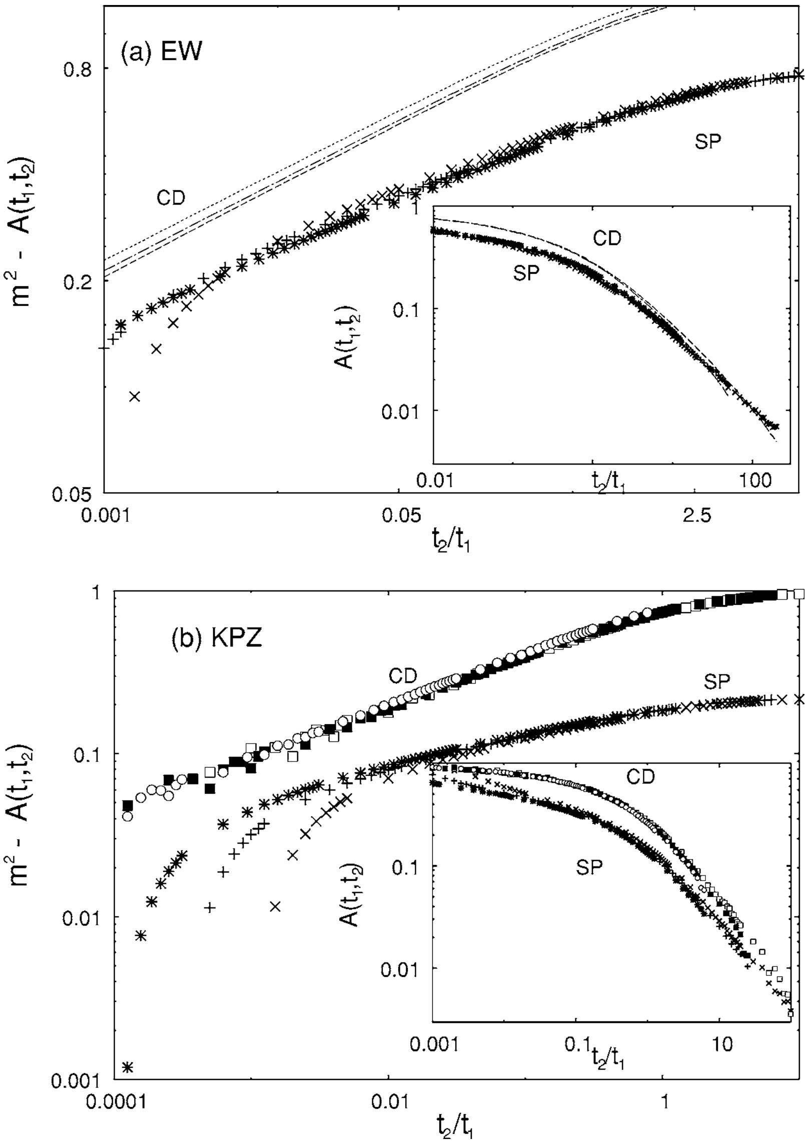

FIG. 3. Aging autocorrelation for CD and SP models with (a) EW and (b) KPZ interfaces. The cusp exponent β' was determined for $t_1 \gg t_2$, after subtraction from m^2. The CD model data in (a) have been multiplied by 1.5 to distinguish them from the SP model data points. The inset shows the power law behavior in the regime $t_1 \ll t_2$. We used $L=2048$ in (a) and $L=8192$ in (b). The inset shows the data with $t_1=500, 2000, 8000$ in both (a) and (b). For extraction of β', we used $t_1 = 2000, 8000, 32\,000$.

$f(\xi)$ [see Eq. (4)], and as τ increases, $g(\xi,\tau)$ is observed to rise and attain a peak [see Fig. 4(a)]. Finally, for larger τ, it decreases and merges with the autocorrelation scaling function $h(\tau)$ [see Eq. (6)]. Note that $G(r=0,t,L) \equiv A(t,L)$ and, by continuity, we expect that when $\xi^z \ll \tau$, the scaling function should behave like $h(\tau)$. From our knowledge of the scaling functions $f(\xi)$ and $h(\tau)$, we have observed that $f(\xi) < h(\tau=\xi^z)$. This implies that $g(\xi,\tau)$ must show an initial rise.

For the CD model on an EW interface,

$$G_{CD}(r,t,L) = \frac{2}{\pi}\sin^{-1}\left[\frac{\sum_{k>0}\dfrac{\exp(-c_k t)2\cos(kr)}{c_k}}{\sum_{k>0}\dfrac{1}{c_k}}\right]. \tag{21}$$

We have evaluated this sum numerically and plotted it against τ, for a fixed value of ξ in Fig. 4(a) (inset), which shows its nonmonotonic nature. In the continuum limit, the argument of arcsine takes the form

$$[2\cos(2\pi\xi) - 2\pi^2\xi + 2\pi\xi Si(2\pi\xi) - 2\pi\nu_1 N(\xi,\tau)]$$

where $N(\xi,\tau)$ is defined as

$$\int_0^\tau dy\sqrt{\pi/\nu_1 y}\,\exp(-\xi^2/4\nu_1 y)[erf(2\pi\sqrt{\nu_1 y} - i\xi/2\sqrt{\nu_1 y}) - 1],$$

which shows explicitly that $G_{CD}(r,t,L)$ is a function of ξ and τ only.

To measure $G_{SP}(r,t,L)$ for particles on an EW surface we performed Monte Carlo simulations as before. After equilibrating the system, we measure $(1/L)\Sigma_{i=1}^{L}\sigma_i(0)\sigma_{i+r}(t)$ for about $L^z/10$ time steps, then after a gap of a few hundred time steps, we take another set of data. We finally average over 10^5 such histories. The results are shown in Fig. 4(a) where we have also included the scaling function $h(\tau)$ to compare the long time behavior. The corresponding results for KPZ surface are shown in Fig. 4(b).

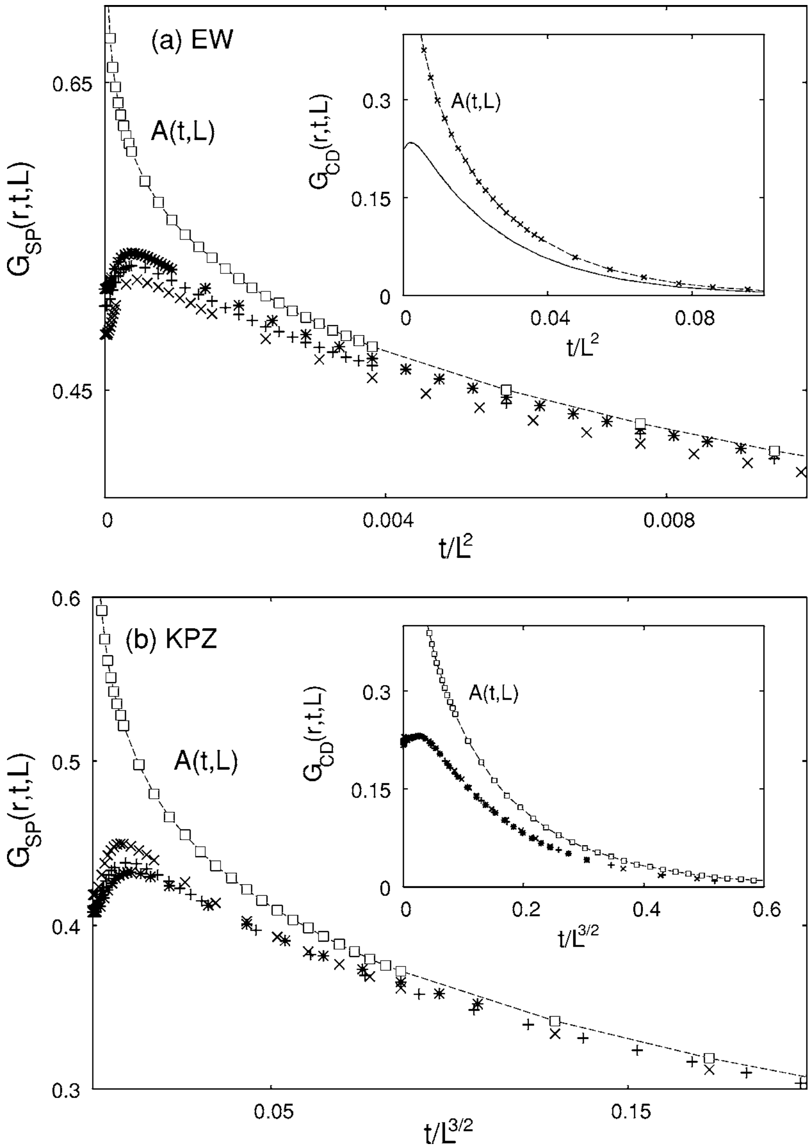

FIG. 4. The time dependence of $G(r,t,L)$ is shown for particles on an (a) EW and (b) KPZ surface for $r/L=0.016$. The values of L are $256,512,1024$ for (a) and $512,1024,2048$ for (b). The scaled autocorrelation is also shown, for comparison. The insets show the same quantity calculated for the corresponding CD model with $r/L=0.125$ for both cases.

VII. LARGEST CLUSTER IN STEADY STATE

One of the key characteristics of FDPO is the presence of strong fluctuations, even in the thermodynamic limit. In the steady state, large clusters are present in the system and the cluster size distribution follows a power law. As a result of fluctuations, these clusters undergo large changes in their lengths, associated with the fact that the macroscopic state of the system keeps changing. For a system of size L, the typical lifetime of a macrostate scales as L^z. The question arises: if the lifetime of a state is so much smaller than exponential, in what sense can we call such a state a "phase?" We have addressed this question in the following way. Let $l_{max}(t)$ be the length of the largest cluster present in the system at time t. In a disordered state, this length scales as $\log L$. But starting from a random initial configuration, as the system approaches steady state, $l_{max}(t)$, although a fluctuating quantity, shows an increasing trend. Finally, in steady state, $l_{max}(t)$ is still fluctuating, thereby changing the macroscopic state of the system. But $l_{max}(t)$ continues to remain substantially above its disordered state value $\log L$. In other words, the system manages to retain its ordered character despite steady-state fluctuations. The system continues to move from one macroscopic state to another over a time scale of L^z. But each of these states is ordered in the sense that they all correspond to large values of $l_{max}(t)$.

We have studied the distribution of l_{max} in steady state as well as in disordered state. Our studies show that as system size increases, the overlap between these two distributions falls off. This means that as L grows, it is increasingly unlikely for the steady state l_{max} to come down as low as its value in a disordered state. The time scale for such an unlikely event would in fact be expected to grow exponentially with L.

After the system has reached steady state, we measure the largest cluster present in that configuration. We let the configuration evolve in time and, after waiting for few hundred time steps, we again measure $l_{max}(t)$. We obtain the distribu-

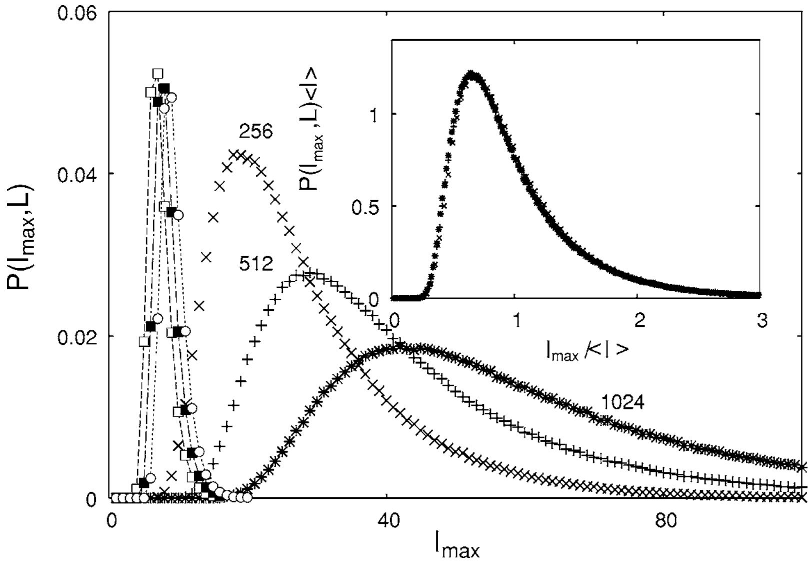

FIG. 5. The distribution of the length of the largest cluster $P(l_{max},L)$ for particles advected by the KPZ surface is shown for $L=256,512,1024$, with the scaling collapse in the inset. The curves to the left show the same distribution in disordered state, after rescaling the y axis by 0.2.

tion $P(l_{max},L)$ after normalizing over 10^6 such data points. $P(l_{max},L)$ for different L values undergo a scaling collapse as l_{max} is rescaled by the mean of the distribution $\langle l\rangle$. We have found that $\langle l\rangle \sim L^{\phi}$, where the exponent ϕ depends on the dynamical rules. For particles on an EW surface $\phi \simeq 0.86$, whereas for KPZ advection, $\phi \simeq 0.60$ while for KPZ antiadvection $\phi \simeq 0.91$. We show the data for KPZ advection in Fig. 5. The disordered state distribution was obtained by averaging over 10^8 data points. The mean of this distribution scales as $\log L$ as mentioned earlier.

It is interesting to ask about other ordered systems in which macroscopic changes occur on a relatively small time scale. One such example is afforded by the zero range process, in which a condensate which holds a finite fraction of particles forms on an arbitrary site, for sufficiently large density. A study of the condensate dynamics shows that it performs an ergodic motion over the entire system [17]. The time scale over which this motion takes place grows as a power of the system size and not exponentially. Thus the macroscopic state of the system changes relatively rapidly, while it continues to remain in the condensed phase. There is, however, an important distinction between this system and the one we have considered. The relative fluctuation in the condensate mass falls to zero with increasing L. On the contrary, in our system, because of the ever-present strong fluctuations, the distribution of the length of the largest cluster remains broad even in the thermodynamic limit.

VIII. DISCUSSION

In this paper, we have studied the dynamics of interacting passive scalars driven by a fluctuating Edwards-Wilkinson or Kardar-Parisi-Zhang surface (or, equivalently, a Burgers fluid), by characterizing the scaling properties of correlation functions in steady state and those of aging correlations during the approach to steady state. It is instructive to compare our results with earlier work on the dynamics of passive scalars in different contexts.

Mitra and Pandit [9] studied the dynamical properties of a system of noninteracting passive particles, advected by an incompressible fluid, whose velocity field is drawn from the Kraichnan ensemble, and therefore has power law correlations in space, but is delta correlated in time. In a Eulerian (space-fixed) framework, they find that the space-time correlation function $G(r,t,L)$ satisfies a diffusion equation in r and t, with an L-dependent diffusion constant. In the quasi-Lagrangian framework (with the origin moving on a Lagrangian trajectory), they obtain $r \sim t^{1/z}$ with $z<2$ and no dependence on L. By contrast, we have studied passive particles with hard-core interactions, advected (in the Burgers case) by a compressible flow which has power law correlations in time. Particles are driven together in our case, rather than spreading out. $G(r,t,L)$ is a function of the scaling combinations r/L and t/L^z, as in a phase-ordered system. However, there appears to be no indication of nontrivial $r-t$ scaling [18]. This difference of behavior reflects the differences between passive scalars with strong clustering or phase-ordering tendencies, and those which spread out in space. In turn, this clustering tendency is presumably a reflection of the strongly compressible nature of the Burgers fluid.

Even when the driving fluid is compressible, the degree of clustering of the passive particles depends on the nature of interactions between them. In the presence of hard-core interactions, the system reaches a phase-ordered state, albeit one with strong fluctuations. As a consequence, in the limit of the small scaling argument, the spatial and temporal correlation functions show a cuspy approach to a finite intercept. However, in the absence of any interaction, the passive particles go into a much more strongly clustered state, where the correlation functions show a power law divergence at the origin [5].

Finally, the study of the largest cluster allows us to arrive at a simple picture of a fluctuation-dominated phase-ordered state. Despite the presence of strong fluctuations, the system never loses its ordered character. Rather, fluctuations carry the system from one ordered configuration to another macroscopically distinct one, over a time scale $\sim L^z$. However, the probability for the system to leave this attractor of ordered states vanishes exponentially with the system size.

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